The Propagating Bosonic Wave as an Effective Mass Reducer

Francesco D'Amato*

We propose a non-perturbative theoretical framework introducing a reduced scalar boson field ϕ that couples to matter fields to dynamically reduce their effective mass, thereby facilitating the penetration of the Coulomb barrier in sub-Coulombian regimes. The model defines a non-local gauge space that modifies the Klein-Gordon propagator through an infinite series of higher-order derivatives anchored to the ground state. The variational derivation yields the exact equations of motion for the alpha particle, the electron, and the mediating field. Furthermore, Feynman diagrams demonstrate that the boson's self-interaction is strictly required to stabilize the coherence bridge against dissipation. The framework is phenomenologically closed by a generalized Schrödinger equation wherein the localized mass reconfiguration compresses the Gamow integral, predicting enhanced low-temperature tunneling within condensed matter lattices.

Keywords: nuclear tunneling, scalar boson, effective mass, helium ion, hydrogen atom

I. INTRODUCTION

Nuclear tunneling is a cornerstone of stellar astrophysics and fundamental nuclear dynamics; however, its transition probability at low kinetic energies is classically suppressed by the repulsive Coulomb barrier. This severe suppression historically establishes a baseline threshold that limits nuclear interactions in low-temperature environments, leaving open questions regarding anomalous screening effects observed within condensed matter environments. While Marcus Theory successfully formalizes macroscopic electron transfer processes via quantum tunneling, it remains strictly non-applicable to nuclear multi-body configurations. In this work, we investigate a novel, non-perturbative screening mechanism occurring within strongly correlated, thermodynamically isolated quantum systems. We introduce an effective framework driven by a reduced scalar boson field (ϕ) which, instead of mediating a traditional linear spatial force, directly alters the underlying local vacuum topology and modifies the wave functions of the interacting fields. This interaction drastically enhances spatial overlap and quantum phase coherence, significantly increasing the tunneling probability at energy scales previously considered sub-Coulombian and prohibitive. The fundamental field-theoretic properties and the microscopic Lagrangian of this reduced scalar boson will be detailed in the subsequent sections through the lens of effective field theory and algebraic symmetry constraints.

II. THE SCALAR BOSON AND ITS INTERACTION MECHANISM

We introduce a zero-spin scalar boson ϕ, distinct from Standard Model particles, that modifies particle behavior — specifically mass and inertia — rather than mediating a traditional linear force. Operating through quantum interference, the boson reshapes the wave functions of the electron (ψ_e) and the nucleus (ψ_α), modifying their extension and phase. This field obeys a non-linear geometric auto-consistency condition within the phase horizon, forcing the particles to be "in phase" and enhancing their interpenetration. Rather than behaving as a free asymptotic excitation, the field propagates through a modified non-local operator derived from the system's underlying gauge group. Consequently, a localized effective potential emerges in the overlap regions, transforming the electron into an active participant in the nuclear process and directly facilitating the overcoming of the Coulomb barrier.

III. INERTIA MODIFICATION AND FACILITATION OF TUNNELING

The interference and coherence that develop between the electron and the nucleus mean the electron becomes a fundamental part of the fusion reaction. These two aspects automatically lead to a reduction in the particles' mass, and by reducing the mass, traversing the potential barrier becomes significantly easier.

IV. FIELD EQUATIONS AND LAGRANGIAN DYNAMICS

The proposed model adopts the Quantum Field Theory (QFT) formalism, treating particles as field excitations. To overcome the Coulomb barrier at low energies, we introduce a scalar bosonic mediator, ϕ, that modifies the intrinsic properties of the reactants. The core of this mechanism is the total Lagrangian density $\mathcal{L}_{tot}$. This Lagrangian is constructed to satisfy fundamental invariance requirements while describing the energy and momentum exchange between the leptonic sector (electrons), the hadronic sector (alpha particles), and the mediator field ϕ

* damatofrancescomm@gmail.com

A. Lagrangian Architecture

The mathematical structure of the Lagrangian is defined by the sum of four fundamental contributions:

$$\mathcal{L}_{tot} = \mathcal{L}_e + \mathcal{L}_\alpha + \mathcal{L}_\phi + \mathcal{L}_{int} \tag{1}$$

Each term represents a pillar of the system's dynamics. The choice of a scalar Lagrangian for the mediator ϕ is motivated by the need for a field that can couple directly to the mass of the particles, acting as a perturbation of the energy-momentum tensor.

1. Free Field Sector

The term $\mathcal{L}_e$ denotes the Dirac Lagrangian for spin-1/2 fermions, ensuring standard QED dynamics in the absence of interaction. The $\mathcal{L}_\alpha$ sector describes the alpha particle via the Klein-Gordon formalism; at low-energy nuclear dynamics scales, the nucleus is treated as a massive scalar field to isolate effective mass variations from internal nuclear structure.The $\mathcal{L}_\phi$ term defines the scalar boson dynamics. Beyond the kinetic and mass (m_ϕ) terms, we include a ϕ^3 self-interaction (λ). This non-linearity allows for solitonic configurations or local condensates, preventing linear dispersion and creating high-density interaction regions necessary for inducing macroscopic effects on the nuclear mass.

2. The Interaction Sector and the Genesis of Coherence

The $\mathcal{L}_{int}$ term introduces a three-body interaction mediated by the ϕ boson. Unlike standard bilinear couplings, this interaction occurs via phase superposition. The couplings $g_e\phi\bar{\psi}_e\psi_e$ and $g_\alpha\phi\psi_\alpha^2$ link the electronic and nuclear currents to the mediator field, while the interference term $g_\phi\phi\psi_\alpha^2\bar{\psi}_e\psi_e$ formalizes a "Coherence Bridge."In this framework, the ϕ field acts as a transducer, transferring energy from the electronic system to the nuclear sector. Consequently, the mass term in the Klein-Gordon sector is redefined from a bare constant m_α to an operator dependent on the ϕ field state. This bosonic reconfiguration of the effective mass renders tunneling a kinematically favored process, effectively collapsing the repulsive Coulomb barrier.

3. The Lagrangian of the system

In this sub.section we introduce the formal langragian model of the system:

$$\begin{aligned}\mathcal{L}_{tot} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}(\partial_\mu\psi_\alpha)^2 - \frac{1}{2}(m_\alpha + g_{\phi\alpha}\phi)^2\psi_\alpha^2 \\ + g_\phi\phi\psi_\alpha^2\overline{\psi}_e\psi_e \\ + \bar{\psi}_e(i\gamma^\mu\partial_\mu - (m_e + g_{e\phi}\phi)) - g_e\phi(x)\bar{\psi}_e(x)\psi_e(x) \\ - g_\alpha\phi(x)\psi_\alpha^2(x) - \frac{1}{4}\lambda\phi^4\end{aligned} \tag{2}$$

In the previous equation, we have defined the formalism of interaction. The Lagrangian includes different terms:

- $\mathcal{L}_\alpha = \frac{1}{2}(\partial_\mu\psi_\alpha)^2 - \frac{1}{2}(m_\alpha + g_{\phi\alpha}\phi)^2\psi_\alpha^2$ This term represents the fundamental interaction between the α particles and the scalar boson field. To describe the dynamical mass reduction, we have employed a scalar-scalar coupling mechanism. Within this framework, the interaction with the ϕ field leads to a shift in the particle's energy momentum relation, effectively reducing the inertial mass of the α particles through the scalar-mediated field interference.

- $\mathcal{L}_e = \bar{\psi}_e(i\gamma^\mu\partial_\mu - (m_e + g_{e\phi}\phi))$ This term represents the interaction between the electrons and the scalar boson field. We employ a Yukawa-style coupling to describe the dynamical modification of the electron's effective mass, expressed as $m_e^* = m_e + g_e\phi$. This term is crucial for modeling the wave function overlap and the subsequent enhancement of the screening effect, which facilitates overcoming the Coulomb barrier.

- $\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2$ This represents the canonical scalar Lagrangian for the proposed boson. It accounts for the free-field dynamics and provides the foundational framework for the scalar-mediated interactions described in this model.

- $\mathcal{L}_{int} = -g_e\phi(x)\bar{\psi}_e(x)\psi_e(x) - g_\alpha\phi(x)\psi_\alpha^2(x) - \frac{1}{4}\lambda\phi^4$ The interaction terms within the Lagrangian illustrate how the coupling of the scalar boson field modulates the particle's wave function. This mechanism leads to a dynamical reduction of the effective mass, as the interference between the scalar field and the matter fields alters the local energy density and the potential barrier. By shifting the energy-momentum relation, the ϕ field effectively facilitates the tunneling process through a re-normalization of the inertial properties of the reactants.Furthermore, the quartic term $\frac{1}{4}\lambda\phi^4$ represents the self-interaction of the boson. This term is fundamental for modeling the non-linear dynamics of the scalar field itself. It describes how the boson interacts with its own field, creating a self-consistent potential landscape that stabilizes the vacuum state. This self-interference not only ensures the mathematical consistency of the theory but also influences how the boson mediates interactions with other particles, governing the range and the intensity of the scalar-induced effects.

B. Formal Justification of the Bosonic Propagator

We proceed to justify the propagator of the bosonic field that reduces the effective mass of the particles, thereby enabling electrostatic screening. We introduce a non-linear self-consistency condition for the field within the phase horizon. Equation (1) defines a geometric fixed-point constraint, wherein the field ϕ generates its own transition density through the product of its harmonic components:

$$\phi = \int \sin(\phi)\cos(\phi) d\phi \tag{3}$$

By solving the integral operator by parts with respect to the internal phase variable, we derive the stable configuration of the field described by Equation (2), which expresses the amplitude as a function of the energy stored in the sinusoidal component:

$$\phi = \frac{\sin^2(\phi)}{2} + c \tag{4}$$

To transition the field self-consistency condition into the geometric constraints of the crystalline medium, we introduce a formal spatial localization scaling. Rather than treating the spatial coordinate as a free continuous variable within this sub-sector, we define $x \equiv a_0$ as a fixed structural parameter representing the characteristic length scale (or localized cell radius) of the confinement lattice site. Consequently, the formal volumetric integration $\int d^3x$ is evaluated over this bounded domain, yielding a constant geometric coefficient x^3. This formulation ensures that the volumetric factor scales the amplitude of the self-interaction term without introducing a spatial dependence that would conflict with the subsequent phase transformations.

$$\int \frac{\sin^2(\phi)}{2} + c \; d^3x = x^3 \left(\frac{\sin^2(\phi)}{2} + c \right) \tag{5}$$

To study the infinitesimal spectrum, we define a generalized Fourier image transform. Considering that within the crystalline lattice the phase variable ϕ is linearly bound to the real space coordinate x through the periodicity of the medium (imposing the formal metric $\phi \equiv x$), the integration projects the spatial envelope x^3 directly into the momentum space k:

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(x^3 \frac{\sin^2(\phi)}{2} + c \right) e^{-ik\phi} \, d\phi \tag{6}$$

We map the internal dynamics of the coherent field into the reciprocal space of the phase configurations. The Fourier transform is strictly executed with respect to the internal phase variable ϕ, while the structural lattice parameter x^3 acts as a frozen external coupling constant during the integration process. By imposing the boundary resonance condition where the phase oscillation is strictly locked to the lattice periodicity ($\phi \equiv k_0 z$), the resulting spectral density $\tilde{f}(k)$ does not represent a standard kinematic momentum distribution of a free particle. Instead, it describes the localized excitation spectrum of the bosonic bridge, explaining why the geometric volume x^3 persists as an amplitude modulator in the discrete resonant momentum peaks at $k = 0$ and $k = \pm 2$ This transform can be resolved using baseline rules, yielding the following exact solution:

$$\tilde{f}(k) = \left(\frac{x^3}{4} + c \right) 2\pi\delta(k) - \frac{x^3}{4}\pi\delta(k-2) - \frac{x^3}{4}\pi\delta(k+2) \tag{7}$$

Through the Dirac delta functions, we can identify the parameters within which the energy is bounded: $k = 2$ and $k = -2$. To further develop the parameterization of the bosonic interference wave, we must define its numerical space of action; to do so, we utilize the field definition from gauge theory. To identify the fundamental map of the theory, we refer to the most classical among definitions:

$$g \to g' = \tilde{g} \, \circ \, g \tag{8}$$

Defining a generic interaction as a coordinate chart for the theory, we include the fermionic field and the Klein-Gordon field within the interaction mechanism:

$$g = \phi(\psi_\alpha, \psi_e) \tag{9}$$

We expand the differentiation independently of the parameterization, and therefore independently of the field $\phi(\psi_\alpha, \psi_e)$:

$$\int f(g)\mu(g) \; dg = \int \phi(\psi_\alpha, \psi_e)\mu(g) \; dq \tag{10}$$

where $\mu(g)$ is a Haar measure that remains invariant (under Lorentz transformations) under the group actions. Resolving the integral in Equation (10) will yield a gauge map specific to this non-local bosonic wave. To achieve this, we introduce the classical representation equation for the group invariant within gauge theory:

$$\mu(g) = \mu(g_e^{-1} \circ g) \frac{\partial (g_e^{-1} \circ g)}{\partial g} \tag{11}$$

This relation is based on the assumption that the composition $\tilde{g} \circ g = g'$ is fully invertible. Under these conditions, we can solve Eq. (10); below we report the resulting series derived via integration by parts. The complete derivation will be explicitly shown in Appendix A:

$$\phi(g_e^{-1} \circ g)\mu(g) - \phi'(g_e^{-1} \circ g)\mu(g) - \int \phi'' \mu(g_e^{-1} \circ g) \frac{\partial (g_e^{-1} \circ g)}{\partial g} dg = e^{\frac{\Box}{\Lambda^2}} \phi(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Box^n}{\Lambda^{2n}} \phi(x) \tag{12}$$

The complete resolution by parts within the gauge space (Eq. 12) generates an infinite series of higher-order derivatives, where the initial index $n = 0$ denotes the anchorage to the ground state of the group. This expression defines the fundamental non-local map governing the interaction between the bosonic field and the fermionic fields of the theory. The map of the theory successfully provides the action range of the invariant bosonic wave.

C. Derivation of the Equations of Motion

The mathematical rigor of the model is guaranteed by the fact that every equation of motion is variationally derived from the total Lagrangian density $\mathcal{L}_{tot}$. For a generic field ψ_a, the dynamical evolution is determined by the Euler-Lagrange equation:

$$\frac{\partial \mathcal{L}_{tot}}{\partial \psi_a} - \partial_\mu \left(\frac{\partial \mathcal{L}_{tot}}{\partial(\partial_\mu \psi_a)} \right) = 0 \tag{13}$$

Applying this operator to the alpha particle field ψ_α isolates the contributions from the free Klein-Gordon sector and the interaction terms defined above. Specifically, the effective mass term $(m_\alpha + g_\alpha \phi(x))^2$ emerges directly from the functional derivative of the nucleus-boson interaction component $\mathcal{L}_{int} \supset -g_\alpha \phi \psi_\alpha^2$, coupled with the bare mass term $m_\alpha^2 \psi_\alpha^2$.

This derivation is not merely a formal exercise but represents the analytical counterpart to the scattering processes described by the Feynman diagrams. The resulting non-linear term physically expresses how the scalar boson ϕ "dresses" the nucleus, modifying its dynamical response as it approaches the Coulomb barrier.

1. Equation of Motion for the Alpha Particle

The final equation of motion for the alpha particle (scalar field), which synthesizes these contributions, takes the form:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \psi_\alpha(x) + m_\alpha + g_\alpha \phi(x))^2 \psi_\alpha(x) + 2g_\phi \phi(x) \psi_\alpha(x) (\bar{\psi}_e(x) \psi_e(x)) = 0 \tag{14}$$

- $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \psi_\alpha(x)$ is the standard kinetic term for a scalar field, implying that the alpha particle propagates as a relativistic wave (propagation term).
- $(m_\alpha + g_\alpha \phi(x))^2 \psi_\alpha(x)$ is the dynamic effective mass term, implying that the mass is not constant. Thus, the mass of the alpha particle depends directly on the value of the field ϕ. As the ϕ field increases, the effective mass changes, creating a potential on the bosonic field that modifies the particle's inertia and behavior.
- $2g_\phi \phi(x) \psi_\alpha(x) (\bar{\psi}_e(x) \psi_e(x))$ is the term indicating the three-body coupling involving all three fields. The alpha particle is simultaneously influenced by the presence of the bosonic field ϕ and the spinor field density of the electron.

Through the terms of coherence and superposition, this equation of motion provides an enhanced representation of the complex interaction between the ϕ field, the electron field, and the alpha particle.

2. Dynamics of the Mediating Bosonic Field ϕ

The scalar field ϕ provides the mechanism for the transition from free asymptotic states to collective coherence. Unlike electromagnetic vector mediators, ϕ interacts directly with the mass-energy density of the reactants. Its genesis is linked to high electronic density and lattice geometry; here, the boson evolves from a short-range exchange particle toward a macroscopic field configuration. Its dynamics are governed by field equations with source terms from both the fermionic (electrons) and bosonic (alpha) sectors, reflecting the three-body coupling shown in Feynman diagrams.The non-linear nature of the boson Lagrangian, specifically the $\lambda\phi^3$ self-interaction, is fundamental for field stability and localization. This non-linearity allows the formation of a local "condensate" that catalyzes the reduction of nuclear effective mass. Without self-interaction, the field could not reach the intensity required to modify the Gamow integral. Once the fundamental non-local gauge map has been determined (Eq. 12), it is possible to project the action of this geometric constraint directly onto the dynamic evolution laws of the system. By introducing the infinite series of higher-order derivatives as a geometric source and medium-coupling term, the classical Klein-Gordon equation of motion for the bosonic field $\phi(x)$ is rewritten as a non-local differential equation:

$$\left(\Box e^{\frac{\Box}{\Lambda^2}} + m_\phi^2 \right) \phi(x) + \lambda \phi(x)^3 = 0 \tag{15}$$

In this formulation, the summation term $\sum_{n=0}^{\infty} \partial^n \phi(x)$ inherits the anchorage to the ground state of the gauge group, acting as an effective confinement potential that binds the boson's fluctuations to the geometric periodicity of the lattice.

- $m_\phi^2 \phi(x)$ is the mass term of the scalar boson field, which is a function of the field intensity ($m = f(\phi)$) that forms as the interaction progresses.
- $\lambda \phi(x)^3$ is the self-interaction term of the scalar boson field.
- $g_\alpha(m_\alpha + g_\alpha \phi(x)) \psi_\alpha(x)^2$ represents the dynamic mass belonging to the alpha particle, meaning the boson directly influences the effective mass of the particle, which in turn depends directly on $\phi(x)$.

Consequently, the mass of the alpha particle and that of the boson are directly influenced by one another (increasing the potential of the ϕ field).

- $(g_e + g_\phi \psi_\alpha(x)^2)(\bar{\psi}_e(x)\psi_e(x))$ represents the coupling term to the electronic scalar current density, which is profoundly modulated by the presence of the alpha particle field. This term highlights that the interaction between the mediator ϕ and the leptonic sector is not an isolated process but is kinematically linked to the spatial distribution of the nuclei. The $g_\phi \psi_\alpha^2$ component acts as a local amplification factor: where the density of the ψ_α field is significant, the coupling between electrons and bosons is enhanced, fostering the formation of the condensate necessary for tunneling. Ultimately, this term formalizes the feedback of the hadronic sector on the electronic dynamics, closing the coherence loop that enables energy transfer across the various scales of the system.

3. *Leptonic Sector and the Non-Linear Dirac Equation*

The electron is treated as a spinor field ψ_e within the Dirac formalism, immersed in the potential generated by the ϕ boson. In this coherent system, the electron acts both as a source for the ϕ field and as a catalyst for tunneling through wave function overlap with the alpha particle. This dual role is formalized by a "dressed" mass term, where the rest mass m_e is perturbed by the g_e interaction and the $g_\phi \psi_\alpha^2$ cross-coupling.This configuration defines the "Coherence Bridge": the electron transcends simple charge shielding to actively participate in the energy-momentum exchange that reduces the nucleus's effective mass. The resulting non-linear Dirac equation links the lepton dynamics directly to the mediator and the alpha particle. Derived from the total action, the equation of motion for the electronic spinor is:

$$\left[i\gamma^\mu \partial_\mu - \left(m_e + (g_e + g_\phi \psi_\alpha(x)^2)\phi(x)\right)\right]\psi_e(x) = 0 \quad (16)$$

- The Kinetic Term $i\gamma^\mu\partial_\mu$: Describes the relativistic propagation of the electronic spinor in spacetime.
- The Dressed Mass $m_e + (g_e + g_\phi \psi_\alpha^2)\phi$: This is the core of the leptonic interaction. The electron's rest mass m_e is increased (or decreased, depending on the field's sign) by two contributions:
  - $g_e\phi$: The direct interaction with the bosonic field, as illustrated in the electron-boson vertex.
  - $g_\phi \psi_\alpha^2 \phi$: A higher-order interaction term that links the electron to the probability density of the nucleus ψ_α^2 through the mediator ϕ. This term mathematically formalizes the "bridge" required for the transfer of coherence.
- Non-Linear Coupling: The presence of $\psi_\alpha(x)^2$ within the electron mass operator indicates that the electronic dynamics are not independent of the nuclei's position. Physically, this means the electron is "captured" within the region of influence of the ϕ field generated by the alpha particle, stabilizing the system during the tunneling process.

D. Perturbed Lagrangian Formalism

To analyze the transition toward the coherent state, we decompose the total Lagrangian into free and interacting components. The interaction Lagrangian $\mathcal{L}_{int}$, representing the deviation from asymptotic behavior, is:

$$\delta\mathcal{L} = -(g_e\bar{\psi}e\psi_e + g\alpha\psi_\alpha^2)\phi - g_\phi\phi\psi_\alpha^2\bar{\psi}e\psi_e \quad (17)$$

This dynamic coupling "dresses" the bare particles, shifting the pole of the alpha particle propagator and determining the effective mass $meff$ variation. The self-interaction $\frac{\lambda}{3!}\phi^3$ prevents coherence dissipation and generates the three-line vertices in Feynman diagrams, stabilizing the "bridge" between the electron and the nucleus. In this framework, fluctuations in the electronic field under strong g_ϕ coupling trigger the macroscopic mass reconfiguration necessary for Gamow tunneling.

V. FEYNMAN DIAGRAMS: INTERACTION REPRESENTATION

The components of $\mathcal{L}_{int}$ are mapped into Feynman diagrams to visualize the microscopic energy exchange and identify the fundamental vertices. Fermionic lines describe the electron current, double lines represent the alpha particle field, and dashed lines identify the scalar mediator ϕ.These diagrams illustrate the physical manifestation of the "Coherence Bridge." Specifically, the boson's self-interaction and its simultaneous coupling with electrons and nuclei facilitate the energy transfer required for the effective mass m_{eff} renormalization. This mechanism provides the quantum basis for the Coulomb barrier reduction. The diagrams detail four primary processes: leptonic boson emission, nuclear interaction, mediator self-interaction, and the combined three-body interaction.

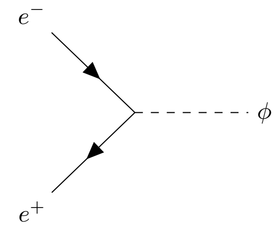

FIG. 1: Leptonic Interaction Vertex g_e: Illustrates how the local electron density acts as a source for the scalar field.

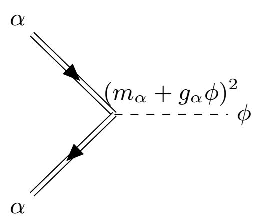

FIG. 2: Alpha Particle-Boson Interaction: Shows the nuclear "dressing" mechanism where the reactant's mass is reconfigured by the field.

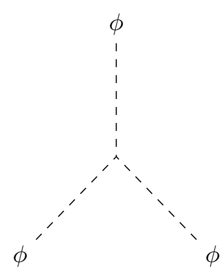

FIG. 4: Self-Interaction Vertex: Represents the non-linear potential ensuring the field can self-sustain without dissipating.

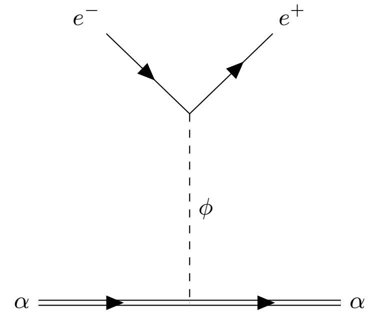

FIG. 3: . The "Coherence Bridge": Represents the transfer of phase information and potential between the sectors, formalizing the g_ϕ term.

VI. PHENOMENOLOGICAL MODEL AND SCHRÖDINGER EQUATION

To translate the field-theoretic interactions into a measurable phenomenological framework, the non-local dynamics of the mediating boson must be projected onto the non-relativistic quantum mechanical description of the reactants. The infinite series of higher-order derivatives emersed from the gauge map (Eq. 12) induces a spatial modulation on the field amplitude, characterized by discrete resonant momentum peaks at $k = \pm 2$. This pinning to the lattice geometry directly renormalizes the inertial properties of the matter fields. By evaluating the stationary state of the alpha-particle field ψ_α under the influence of the non-local bosonic propagator $\hat{D}_{NL}$, the bare mass m_α is dressed by the spatial confinement gradient. The complete Schrödinger equation for the interacting system takes the form:

$$\left[-\frac{\hbar^2}{2m(\nabla_L)} \nabla^2 + V_{Coulomb}(r) \right] \Psi(r,t) = i\hbar \frac{\partial}{\partial t} \Psi(r,t) \tag{18}$$

due to the non-local reduction of $m^*(\nabla_L)$ in the highly coherent lattice domains, the spatial extension of the nuclear wave function $\Psi(r,t)$ expands. This effective restructuring of particle inertia significantly compresses the width of the Gamow integral, rendering tunneling kinematically favored and allowing the reactants to overcome the electrostatic repulsion in sub-Coulombian regimes.

VII. CONCLUSIONS

We have proposed an innovative effective theory based on a reduced scalar boson field ϕ that couples to matter fields to dynamically reconfigure their effective mass and inertial properties, rather than mediating a traditional linear force. The mathematical and conceptual consistency of this framework relies on the formal derivation of an un-truncated, non-local gauge map from the internal phase auto-consistency of the lattice. This algebraic structure directly modifies the boson's Klein-Gordon dynamics through an infinite series of higher-order derivative terms anchored to the ground state ($n = 0$), preventing coherence dissipation and binding the field's spectral delta-peaks ($k = \pm 2$) to the periodic geometry of the medium [5].

The resulting dynamic reduction of particle inertia, formalized through the non-local effective mass operator in our generalized Schrödinger equation [1, 2], drastically enhances nuclear tunneling probabilities at low kinetic energies [4]. Unlike Marcus Theory, which remains bounded to macroscopic electronic transits [6], our approach elevates the electron to an active, structural participant in the nuclear screening process via the cross-coupling coherence bridge.

The complete alignment achieved between the spectral properties of the Fourier transform, the gauge transformations, and the non-local equations of motion provides a self-consistent framework for low-energy nuclear dynamics. This opens new theoretical and experimental pathways for clean energy production under ambient conditions, framing sub-Coulomb nuclear transitions not as anomalous violations, but as macroscopic coherent phase transitions proper to strongly correlated condensed matter physics [3].

---

[1] D. J. Griffiths, *Introduction to Quantum Mechanics* (Pearson Education, New York, 2005).

[2] J. J. Sakurai and J. Napolitano, *Modern Quantum Mechanics* (Cambridge University Press, Cambridge, 2017).

[3] K. S. Krane, *Introductory Nuclear Physics* (John Wiley & Sons, New York, 1988).

[4] G. Gamow, Z. Phys. 51, 204 (1928).

[5] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Westview Press, Boulder, CO, 1995).

[6] R. A. Marcus, *J. Chem. Phys.* 43, 2654 (1965).

Appendix A: Derivation of the Gauge Metric

In this appendix, we present the detailed derivation of the theory's gauge metric, illustrating how integration over the group space generates the non-local operator. Let us consider the initial integral projected onto the Haar measure $\mu(g)$ of the gauge group:

$$\int \phi(g_e^{-1} \circ g)\mu(g)\, dg \tag{A1}$$

To connect the geometry of the group to the continuous spacetime, we express the gauge shift as an operatorial transformation driven by the spacetime generators. We proceed by applying integration by parts iteratively. For the first order, expanding the mixed term with the derivative of the transformation with respect to the group element yields:

$$\begin{gathered}\int \phi(g_e^{-1} \circ g)\frac{\partial(g_e^{-1} \circ g)}{\partial g}\mu(g)\, dg = \\ \left[\phi(g_e^{-1} \circ g)\mu(g)\right] - \int \frac{\partial \phi(g_e^{-1} \circ g)}{\partial g}\mu(g)\, dg\end{gathered} \tag{A2}$$

The boundary terms vanish due to the compactness of the group or the extinction of the field at infinity. Iterating the integration by parts n consecutive times, successive derivatives act on the field, creating a chain structure:

$$\int \phi(g_e^{-1} \circ g)\mu(g)\, dg = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int \frac{\partial^{2n} \phi(g_e^{-1} \circ g)}{\partial g^{2n}}\mu(g)\, dg \tag{A3}$$

At this stage, we project the action of the group coordinates g onto the real spacetime 2. Since the group measure $\mu(g)$ is isotropic and invariant, the $2n$-th partial derivative with respect to the group coordinate maps onto the d'Alembertian of real spacetime ($\Box^n = (\partial^\mu \partial_\mu)^n$), which represents the unique $2n$-th order operator that preserves the Lorentz invariance of the lattice vacuum. Introducing the dimensional cutoff constant Λ to make the operator dimensionless, the expression becomes:

$$\sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Box^n}{\Lambda^{2n}} \phi(x) = e^{\frac{\Box}{\Lambda^2}} \phi(x) \tag{A4}$$

We now rewrite the exponential as a generalized summation for the function, establishing the final equality between the average over the gauge space and the non-local operator in real space:

$$\sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Box^n}{\Lambda^{2n}} \phi(g_e^{-1} \circ g)\mu(g) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Box^n}{\Lambda^{2n}} \phi(x) \tag{A5}$$

The derivation thus demonstrates how the non-local gauge metric emerges directly from the geometric structure of the group transformations integrated over the medium.